Performance study of DD ML Phase Estimators for DS-CDMA Communications Systems

L.Schumacher*
Center for PersonKommunikation
Fredriks Bajers Vej, 7A5
DK-9220 Aalborg East, Denmark
schum@cpk.auc.dk

L. Vandendorpe
Communications and Remote Sensing Lab
Place du Levant, 2
B-1348 Louvain-la-Neuve, Belgium
vandendorpe@tele.ucl.ac.be

ABSTRACT

In the case of Data-Aided (DA) Maximum-Likelihood (ML) phase estimators operating in Direct Sequence-Code Division Multiple Access (DS-CDMA) communication systems, it has been shown [1, 2] that a multiuser (MU) design of the estimator helped to define parameter estimators exhibiting a lower variance than those designed in the conventional, single-user (SU) way. The present paper applies the same MU design strategy in the more realistic case of Decision-Directed (DD) estimators. The performance of these DD structures are derived from the DA variance expressions, assuming decisions to be correct and limiting the mitigation of the Multiple Access Interference (MAI) to its causal contribution.

1 Introduction

The ever growing popularity of spread-spectrum techniques in multiple access communication systems has led to the demonstration of the performance limitations of conventional receiver structures [3]. This has been first illustrated at the detection stage. Parallel and serial interference cancelers have been introduced to mitigate the MAI inherent to the use of DS-CDMA schemes [4, 5]. The same limitations also plague the estimation stage. Like the symbols detectors, the parameter estimators suffer from MAI. However, it has been demonstrated in [2] for Feedback (FB) implementations and in [1] for Feedforward (FF) structures that the incidence of MAI on conventional phase estimators is due to a design omitting the presence of the other active users. In circumstances where knowledge about all the users active in the system can be gathered, for instance during a training period on the uplink of a mobile DS-CDMA communication system, a parameter estimator which relies on the information related to only one user, neglecting all the other ones, is clearly not optimal. MU phase estimators exploiting the total information have thus been introduced. It has been shown in the DA case that such a design strategy leads to structures mitigating the incidence of MAI, which exhibit a lower parameter variance than estimators designed in a SU perspective.

How interesting the estimation structures mentioned here above might be, considering only DA estimators, which require training periods, is not realistic. It is much more meaningful to build DD estimators according to the same design philosophy. However, the performance study of such structures is rather intricate. Exact analytical results obtained in the case of SU communication systems in Additive White Gaussian Noise (AWGN) channels are presented in [6]. This paper presents approximated analytical performance results for MU DD ML FB and FF phase estimators working with Binary Phase Shift Keying (BPSK)-modulated signals. These results are derived from previously published performance study of DA structures.

2 System description

Consider the uplink of a coherent DS-CDMA communication system accommodating N_u users. The low-pass equivalent received signal at the Base Station (BS), sum of the contributions of the N_u active users, can be written as

$$r(t) = \sum_{k=1}^{N_u} \sqrt{2E_k}\, e^{j\phi_k} \sum_{m=-\infty}^{+\infty} I_k^m h_k(t-mT) + n(t) \quad (1)$$

where $E_k\sigma_{I_k}^2$ represents the emitted energy per binary symbol I_k^m, $h_k(t)$ is the complete impulse response for user k, embedding both its spreading waveform and its channel impulse response, T stands for the symbol duration and $n(t)$ is the low-pass equivalent of an AWGN with two-sided power spectral density $\frac{N_0}{2}$.

At the receiving end, the bandpass signal is downconverted to baseband using a local oscillator with correct frequency but arbitrary phase. Hence, ϕ_k, the parameter of interest in this work, appears in (1) as

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the carrier phase difference between transmitter's and receiver's oscillators.

$r(t)$ is fed to channel-matched filters whose outputs are sampled at symbol rate. y_k^p stands for the normalised matched filter output

$$y_k^p = \frac{1}{\sqrt{2E_k T}} \int_{-\infty}^{+\infty} h_k^*(t - pT)\, r(t)\, dt \quad (2)$$

$$= e^{j\phi_k} \sum_{q=-\infty}^{+\infty} I_k^q x_{k,k}^{p-q} + \sum_{\substack{l=1 \\ l \neq k}}^{N_u} e^{j\phi_l} \sqrt{\frac{E_l}{E_k}} \sum_{q=-\infty}^{+\infty} I_l^q x_{k,l}^{p-q} + \nu_k^p \quad (3)$$

where $x_{k,l}^{p-q}$ represents the normalised channel correlation coefficient between users k and l for a time shift $(p-q)T$ and ν_k^p are zero-mean complex samples of the noise filtered by the matched filter.

3 DD ML FB Phase Estimator

In [2], a MU phase recovery loop was introduced in a DA context. This loop tracks the phase ϕ_u according to an error signal $u_{u,DA}^m$

$$u_{u,DA}^m = \Im \left[\begin{array}{l} e^{-j\hat{\phi}_u^m} (I_u^m)^\star y_u^m \\ - \sum\limits_{\substack{k=1 \\ k \neq u}}^{N_u} \sqrt{\frac{E_k}{E_u}}\, e^{j\left(\hat{\phi}_k^m - \hat{\phi}_u^m\right)} \\ \quad \sum\limits_{n=-\infty}^{+\infty} (I_u^m)^\star I_k^n x_{u,k}^{m-n} \end{array} \right]. \quad (4)$$

relying on a perfect knowledge of the transmitted data. It differs from its SU counterpart in that it includes a term mitigating the MAI plaguing the system through the matched filter outputs y_u^m (Second term of (4)).

In order to derive the performance of the loop, and more precisely the variance of the parameter estimate $\hat{\phi}_u$, $u_{u,DA}^m$ is split into its mean value and the loop noise. Using the auto-correlation function of the loop noise, one can derive its power spectral density, filter it through the loop filter and finally compute the variance of the estimate. These steps were followed in [2] and it was shown that the MU DA estimator exhibits a lower variance than the SU one.

The MU DA phase recovery described by relation (4) can be turned into a DD one by substituting decisions $\hat{\mathbf{I}}$ for symbols $\mathbf{I}$. Under some hypotheses, the results presented in [2] in the case of DA ML FB estimators can be extended to illustrate the performance of such DD structures. First, the present extension assumes decisions to be correct. It also requires that the estimators are causal ($n \leq 0$). This restriction to causal structures results from the DD mode. In DA mode, the transmitted message is known from the very beginning. The estimation stage can thus use the knowledge of future symbols with respect to the running time instant. On the contrary, the DD mode can only exploit the knowledge of already detected symbols, since it uses fedback decisions from the detection stage. As a result, DD structures are to be causal.

Limiting the MAI mitigation term to its causal contributions, the auto-correlation function of the DD loop noise writes at equilibrium (estimation error $\mathbf{\Delta} = \mathbf{\Phi}$ - $\hat{\mathbf{\Phi}} = \mathbf{0}$):

$$C_{u,u}^m(\mathbf{0}) = \delta(m) \left\{ \begin{array}{l} \sum\limits_{p=-\infty}^{+\infty} \left[\Im\left(x_{u,u}^p\right)\right]^2 \\ + \frac{N_0 x_{u,u}^0}{2E_u T} \\ + \sum\limits_{\substack{k=1 \\ k \neq u}}^{N_u} \frac{E_k}{E_u} \sum\limits_{p=-\infty}^{+\infty} \left[\Im\left(e^{j\delta_{k,u}} x_{u,k}^p\right)\right]^2 \\ - \sum\limits_{\substack{k=1 \\ k \neq u}}^{N_u} \frac{E_k}{E_u} \sum\limits_{p=-\infty}^{0} \left[\Im\left(e^{j\delta_{k,u}} x_{u,k}^p\right)\right]^2 \end{array} \right\} - \left[\Im\left(x_{u,u}^m\right)\right]^2. \quad (5)$$

where $\delta_{k,u} = \phi_k - \phi_u$. It has been stressed in [2] that the advantage of MU estimators with respect to SU ones lies in the mitigation of the MAI entering the system (third term of (5)) by the last term of the relation. This advantage is partly lost in DD estimators, as the mitigation is limited to the causal part of the MAI ($p \leq 0$). Hence, the DD estimator is plagued by the anti-causal part of the MAI. As a result, one can expect the variance of the DD estimator to be greater than or, at best, equal to the one of the DA estimator.

Indeed, under some conditions (non-dispersive channels for instance), the MAI incidence is mostly concentrated in the $x_{u,v}^0$ channel correlation coefficient. Then, if this one is involved in the mitigation term for DD estimators as it is for DA ones, both estimators exhibit the same variance. On the other hand, the strict limitation of the interference mitigation to $p < 0$ does not enable to cancel it. This leads to a less efficient structure whose variance equals the SU one, as shown in Figure 1[1] in the case of an Inter-Symbol Interference (ISI)-free scenario: dispersive COST 207 Hilly Terrain (HT) channel [7] with small baud rate. Beside the variance of the MU DA ML FB and SU ML FB estimators[2], the variances of two MU DD estimators are plotted. The first DD estimator includes the $x_{u,v}^0$ term in its mitigation contribution (curve "DD

[1] In the figures of this paper, CRLB stands for Cramér-Rao Lower Bound.

[2] The DA/DD distinction is irrelevant for SU structures as they do not implement the mitigating term.

with"), but the second (curve "DD without") does not. Obviously, the latter exhibits poor performance, close to the SU one, whereas the former fits almost the MU DA curve.

Nevertheless, in dispersive environments, the MAI incidence is spread over a span of coefficients $x^m_{u,v}$, $|m| = 0, 1, \ldots$ Those among them which contribute to the anti-causal part of the interference, and are thus not mitigated by any of the two considered DD structures (with and without mitigation of $x^0_{u,v}$), cause an increase of the variance. This is illustrated in Figure 2, where the variances of DA and DD estimators are compared in a dispersive HT channel. The variance floor that limits the performance of the DA estimator as a result of ISI stands below the variance floor related to the DD estimator cancelling the incidence of the term weighted by $x^0_{u,v}$ (curve "DD with"). The difference between them is due to the incomplete MAI mitigation. However, one can notice that, despite missing the MAI due to the present symbol, the second MU DD estimator (curve "DD without") performs slightly better than the SU one, thanks to the mitigation of MAI due to past symbols.

4 DD ML FF Phase Estimator

Considering now FF implementations, the general expression of the estimation error $\Delta_u = \phi_u - \hat{\phi}_u$ of a FF structure was given in [1] as follows:

$$\Delta_u = \frac{\Im\left(e^{-j\phi_u}\ D_u\right)}{\Re\left(e^{-j\phi_u}\ D_u\right)} \tag{6}$$

At first sight, the FF phase estimator appears to be the argument of a complex phasor $e^{-j\phi_u}\ D_u$. Notice however that some linearisation trick has been applied to avoid the $\tan^{-1}$ non-linearity. In the DD case, D_u expands into:

$$\begin{aligned} D_u \quad = \quad & \sum_{m=1}^{N} \left(\hat{I}^m_u\right)^{\star} y^m_u \\ & - \sum_{k=1}^{N_u} \sqrt{\tfrac{E_k}{E_u}}\, e^{j\phi_k} \left[1 + j\left(\hat{\phi}_k - \phi_k\right)\right] \\ & \qquad \sum_{m=1}^{N} \sum_{n=-\infty}^{0} \left(\hat{I}^m_u\right)^{\star} \hat{I}^n_k\, x^{m-n}_{u,k}. \end{aligned} \tag{7}$$

with a similar causal restriction ($n \leq 0$) than in the previous section. With some other minor approximations, this limitation leads to the following expression of the estimation error in the case of the MU DD ML FF estimator:

$$\Delta_u \quad \cong \quad \frac{\left\{\begin{array}{l} \Re\left(MAI^c_{v,u}\right)\ \Im\left(ISI_v + Noise_v\right) \\ -\ \Im\left(ISI_u + MAI^a_{v,u}c + Noise_u\right) \\ \quad \Re\left(Direct_v + ISI_v\right) \end{array}\right\}}{\left\{\begin{array}{l} \Re\left(Direct_u + ISI_u\right)\ \Re\left(Direct_v + ISI_v\right) \\ -\Re\left(MAI^c_{v,u}\right)\ \Re\left(MAI^c_{u,v}\right) \end{array}\right\}} \tag{8}$$

where the respective contribution of the useful term $(x^0_{u,u})$, of the ISI and of the noise write

$$Direct_u \quad = \quad \sum_{m=1}^{N} |I^m_u|^2\, x^0_{u,u} \tag{9}$$

$$ISI_u \quad = \quad \sum_{m=1}^{N} \sum_{\substack{n=-\infty \\ n \neq m}}^{+\infty} \left(I^m_u\right)^* I^n_u x^{m-n}_{u,u} \tag{10}$$

$$Noise_u \quad = \quad e^{-j\phi_u} \sum_{m=1}^{N} \left(I^m_u\right)^* \nu^m_u. \tag{11}$$

One can notice that the MAI contribution is split into its causal $MAI^c_{v,u}$

$$MAI^c_{u,v} = \sqrt{\frac{E_v}{E_u}}\, e^{j(\phi_v - \phi_u)} \sum_{m=1}^{N} \sum_{n=-\infty}^{m} \left(I^m_u\right)^* I^n_v x^{m-n}_{u,v} \tag{12}$$

and anti-causal $MAI^a_{v,u}$ parts

$$MAI^a_{v,u} = \sqrt{E_v} E_u\, e^{j(\phi_v - \phi_u)} \sum_{m=1}^{N} \sum_{n=m+1}^{+\infty} \left(I^m_u\right)^* I^n_v x^{n-m}_{v,u}. \tag{13}$$

Notice that no difference is made in relations (9-13) between decisions and data due to the hypothesis of correct decisions.

From the point of view of the variance, this split of the MAI adds new terms to the variance expressions presented in [1]. A simplified version is obtained by limiting the study to the most relevant one, $\Im\left(MAI^a_{v,u}c\right)\ \Re\left(Direct_v\right)$. Its incidence is a supplementary term whose numerator writes

$$4\, \tfrac{E_v}{E_u} \left(N x^0_{u,u}\right)^2 \left\{ \sum_{m=1}^{N} \sum_{n=m+1}^{+\infty} \left\{ \left|x^{n-m}_{v,u}\right|^2 - \Re\left[e^{2j\,\delta_{u,v}} \left(x^{n-m}_{v,u}\right)^2\right] \right\} \right\}. \tag{14}$$

Its denominator is given by relation (20) in [1].

The term in (14) generates a variance increase independent of the $\frac{E_s}{N_0}$ ratio, whose importance depends on the relative weight of the unmitigated anti-causal part of the MAI. This variance increase graphically translates into a raise of the variance floor (See Figure 3, curve "DD with"). The variance is even greater if the present symbol is not taken into account (curve "DD without"), as already explained in the DA case. However, in dispersive environments such as the one considered in Figure 3, a (slight) improvement with respect to the SU estimator is still noticeable thanks to the mitigation of the MAI due to past symbols.

5 Conclusions

The introduction of the causal restriction of DD ML phase estimators in DA expressions presented in [1, 2] has led to a reinterpretation of these relations in a DD perspective. Under the hypothesis of correct decisions, it has been shown that the variance of the MU DD estimator is greater than, or at best equal to the one of the MU DA estimator, but lower than, or at worst equal to the one of the SU estimator. The quality of the DD estimator mainly depends on the way it deals with the interference related to the present symbol.

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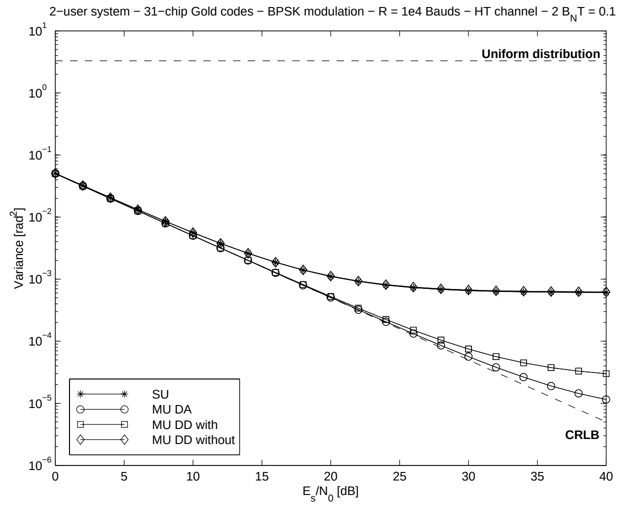

Figure 1: Variance of DD ML FB estimators in ISI-free scenario

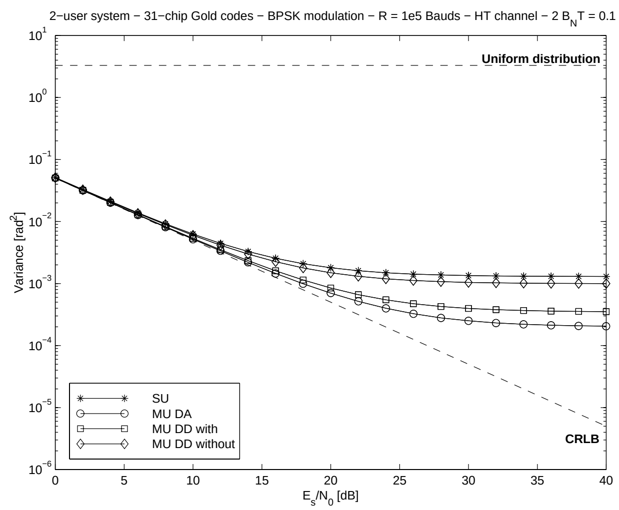

Figure 2: Variance of DD ML FB estimators in presence of ISI

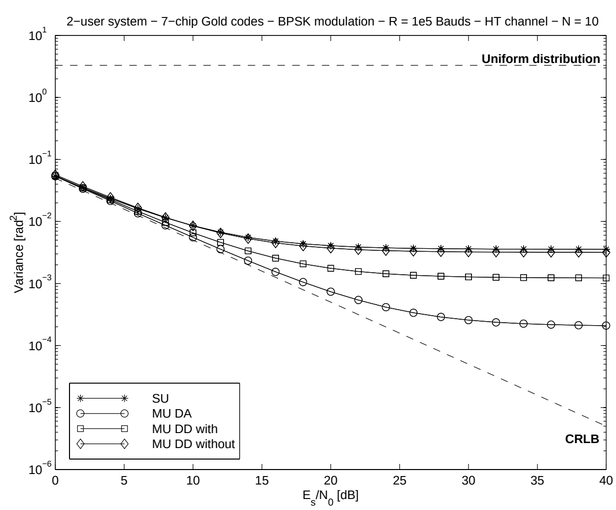

Figure 3: Variance of ML FF estimators in presence of ISI