MAI Mitigation in DA ML Carrier Phase Recovery Loops for DS-CDMA Systems

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Abstract

This paper analytically investigates Maximum-Likelihood (ML) phase recovery in a multiuser system granting access through the Direct Sequence-Code Division Multiple Access (DS-CDMA) technique. A Data-Aided (DA) phase recovery loop is designed to face Multiple Access Interference (MAI). Open- and closed-loop behaviours are studied and illustrated by computations. It is shown that the multiuser phase estimator mitigates MAI and exhibits a smaller variance than the single-user one.

1 Introduction

In DS-CDMA communication systems, Verdú showed that optimal multiuser detection requires prior reliable signal synchronisation in the broad sense (timing, carrier frequency and phase, channel estimation...) [1]. But, in such a multiuser context, efficient detection and parameter estimation structures cannot be driven from a conventional single-user perspective. Indeed, since users of a DS-CDMA system are active at the same time in the same fringe of the spectrum, an MAI component appears at the receiving end as soon as the orthogonality between users is lost. This contribution perturbs the correct working of receivers designed under the assumption that MAI would be negligible. Performance degradation gets even worse when conventional single-user receivers face a Near-Far effect. However, [2] stresses that these receivers are plagued by the Near-Far effect as a result of a design trying to cancel MAI, instead of using the information it bears.

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In order to exploit its information content, MAI has to be taken into account in the design of both detection and estimation processes. Much work has been done in that perspective, as well in the field of detection [3] as in the field of parameter estimation. In the latter, ML estimation appears to be the optimal solution, but its computational complexity is exponential in the number of users. Estimation strategies have been proposed to alleviate this complexity, relying on Extended Kalman Filtering (EKF) [4], Multiple Signal Classification (MUSIC) [5] or Expectation-Maximisation (EM) [6]. However, the performance of these receivers is difficult to establish analytically. Most often, it is illustrated by Monte-Carlo simulations. Only recently appeared a first paper [7] claiming to present an analytical derivation of the effect of MAI on parameter estimation.

The purpose of the present paper is to bring another contribution to the analytical performance study of multiuser parameter estimation in CDMA systems. It is limited to DA phase recovery loops, dealing with BPSK-modulated data symbols. Open-loop and closed-loop studies will be presented. Expressions of the S-curves and correlation properties of the loop noise will be established. The multiuser phase jitter variance will be computed and compared to the variance of single-user recovery loops and to the Cramér-Rao lower bound (CRLB).

2 System description

Consider the uplink of a coherent CDMA communication system accommodating N_u users. The low-pass equivalent signal $x_k(t)$ emitted by user k writes $x_k(t) = \sqrt{2E_k}\sum_{m=-\infty}^{\infty} I_k^m s_k(t - mT)$ where E_k is the emitted energy of user k, T is the symbol duration and I_k^m are the data symbols. $s_k(t) = \sum_{m=0}^{N_c-1} a_k^m u(t - mT_c)$ is the spreading waveform for user k, where $\{a_k^m\}$ is the pseudo-random spreading code, T_c is the chip duration, $N_c = \frac{T}{T_c}$ is the spreading factor and $u(t)$ is a rectangular pulse of duration T_c. Signals are transmitted through channels having impulse responses $c_k(t)$. Defining $h_k(t) = s_k(t) \otimes c_k(t)$, the low-pass equivalent signal received at the Base Station (BS), sum of the contributions of the N_u users, writes

$$r(t) = \sum_{k=1}^{N_u} \sqrt{2E_k}\, e^{j\phi_k} \sum_{m=-\infty}^{+\infty} I_k^m h_k(t-mT) + n(t) \tag{1}$$

Received signal $r(t)$ is assumed to be demodulated to baseband using a local oscillator with correct frequency but arbitrary phase. Thus, ϕ_k is the carrier phase difference between transmitter's and receiver's oscillators and is assumed to be constant. Finally, $n(t)$ is the low-pass equivalent of an additive white Gaussian noise (AWGN) with two-sided power spectral density $\frac{N_0}{2}$.

In the following, the code sequences $\{a_k^m\}$ and the channel responses $c_k(t)$ will be supposed to be perfectly known. Channel responses will be assumed to be static, with power delay profiles defined in accordance with COST 207 models [8]. Timing is perfectly recovered.

3 Multiuser ML phase estimation

The parameters $\{\phi_k\}$ to be estimated are regarded as deterministic but unknown. Optimum estimates are then obtained by applying ML estimation. ML estimates are theoretically derived from the maximization of the likelihood function $\Lambda(\mathbf{r}|\,\mathbf{\Phi})$. A necessary but not sufficient condition for this maximum can be obtained by setting to zero the first derivative of the logarithm of $\Lambda(\mathbf{r}|\,\mathbf{\Phi})$ with respect to the unknown parameters [9]. Due to the multiuser context, one gets as many equations as parameters to be estimated. Each of these equations takes into account the fact that the received signal results from the contribution of several users.

In the assumed multiuser context, the logarithm of the likelihood function $\Lambda_L(\mathbf{r}|\,\mathbf{\Phi})$ writes

$$\begin{aligned}
&\Lambda_L(\mathbf{r}|\,\mathbf{\Phi}) \\
&= \text{Cst} + \sum_{k=1}^{N_u} \frac{2E_kT}{N_0} \Re\left[e^{-j\phi_k} \sum_{m=-\infty}^{+\infty} (I_k^m)^\star y_k^m\right] \\
&\quad - \sum_{k=1}^{N_u} \frac{E_kT}{N_0} \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} (I_k^m)^\star I_k^n x_{k,k}^{m-n} \\
&\quad - \sum_{k=1}^{N_u} \sum_{\substack{l=1 \\ l\neq k}}^{N_u} \frac{\sqrt{E_kE_l}T}{N_0} e^{j(\phi_k-\phi_l)} \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} (I_l^m)^\star I_k^n x_{l,k}^{m-n}
\end{aligned} \tag{2}$$

where $x_{k,l}^{p-q} = \frac{1}{T}\int_{-\infty}^{+\infty} h_k^\star(t-pT)h_l(t-qT)dt$ represents the normalised channel correlation coefficient between users k and l for a time shift $(p-q)\,T$ and $y_k^p = \frac{1}{\sqrt{2E_kT}}\int_{-\infty}^{+\infty} r(t)h_k^\star(t-pT)dt$ stands for the normalised matched filter output

$$\begin{aligned}
y_k^p &= e^{j\phi_k} \sum_{q=-\infty}^{+\infty} I_k^q x_{k,k}^{p-q} \\
&\quad + \sum_{\substack{l=1 \\ l\neq k}}^{Nu} e^{j\phi_l} \sqrt{\frac{E_l}{E_k}} \sum_{q=-\infty}^{+\infty} I_l^q x_{k,l}^{p-q} \\
&\quad + \frac{1}{\sqrt{2E_kT}} \int_{-\infty}^{+\infty} n(t) h_k^\star(t-pT)dt
\end{aligned} \tag{3}$$

The first term of (3) includes as well the useful symbol I_k^p as the ones interfering through Inter Symbol Interference (ISI). The following term represents the MAI contribution.

In the next sections, we will study ML estimation of phase parameters in a Data-Aided (DA) context, considering BPSK modulated symbols.

4 DA ML phase estimation in a multiuser context

Calculating the first derivative of (2) with respect to $\{\phi_k\}$, knowing I_k^p, and setting the results equal to zero leads to a set of N_u conditions of the type

$$\left.\frac{\partial \Lambda_L(\mathbf{r}|\,\mathbf{\Phi})}{\partial \phi_u}\right|_{\mathbf{\Phi}=\hat{\mathbf{\Phi}}} = \frac{2E_uT}{N_0}$$

$$\Im\left[\begin{array}{l} e^{-j\hat{\phi}_u} \sum_{m=-\infty}^{+\infty} I_u^m y_u^m \\ -\sum_{\substack{k=1\\k\neq u}}^{N_u} \sqrt{\frac{E_k}{E_u}}\, e^{j(\hat{\phi}_k-\hat{\phi}_u)} \\ \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} I_u^m I_k^n x_{u,k}^{m-n} \end{array}\right] = \frac{2E_uT}{N_0}\sum_{m=-\infty}^{+\infty} u_u^m = 0 \quad (4)$$

This condition can describe as well feed-forward as feed-back phase recovery implementations. The present paper only deals with feed-back structures working in tracking mode. The loop is driven by the error signal u_u^m. It results from two contributions. Besides the classic one, relying on matched filter outputs y_u^m [10], one notices a second term, only depending on interfering users. This term comes out from the fact that the log-likelihood function $\Lambda_L(\mathbf{r}|\,\mathbf{\Phi})$ takes into account the multiuser context.

4.1 Open-loop study

The first step in the study of a recovery loop in tracking mode is to determine the operating point of the loop, that is to say the position for which the error signal driving the loop will be null. At the operating point, U_u, the expectation of u_u^m in open-loop, is equal to zero. For the system under study, $U_u = K_{D,u}\sin\left(\phi_u - \hat{\phi}_u^m\right) = K_{D,u}\sin\Delta_u$ where $K_{D,u} = x_{u,u}^0$ is the phase detector gain. U_u depends on Δ_u through a sinusoidal function. This means that driving the error signal of the loop to zero ($U_u = 0$) is equivalent to have $\Delta_u = 0$, which is an unbiased operating point. It is not surprising to find the same result than for single-user estimators. In the DA context, MAI is cancelled thanks to the perfect knowledge of interfering users'messages.

4.2 Closed loop study

u_u^m can be split into its mean value U_u and the loop noise ν_u^m, which is the sum of the additive noise

$$\Im\left[\frac{1}{\sqrt{2E_uT}}e^{-j\hat{\phi}_u^m} I_u^m \int_{-\infty}^{+\infty} n(t)h_u^\star(t-mT)dt\right] \quad (5)$$

and the self-noise

$$\Im\left[\begin{array}{l} e^{j(\phi_u-\hat{\phi}_u^m)} \sum_{n=-\infty}^{+\infty} I_u^m I_u^n x_{u,u}^{m-n} \\ +\sum_{\substack{k=1\\k\neq u}}^{Nu} e^{j(\phi_k-\hat{\phi}_u^m)}\sqrt{\frac{E_k}{E_u}} \sum_{n=-\infty}^{+\infty} I_u^m I_k^n x_{u,k}^{m-n} \\ -\sum_{\substack{k=1\\k\neq u}}^{Nu} e^{j(\hat{\phi}_k^m-\hat{\phi}_u^m)}\sqrt{\frac{E_k}{E_u}} \sum_{n=-\infty}^{+\infty} I_u^m I_k^n x_{u,k}^{m-n} \end{array}\right] -K_{D,u}\sin\left(\phi_u - \hat{\phi}_u^m\right) \quad (6)$$

Considering working close to the operating point, U_u is replaced by its linear decomposition around the operating point $\mathbf{\Delta} = \mathbf{0}$

$$\mathbf{U} = \left.\frac{\partial \mathbf{U}}{\partial \mathbf{\Delta}}\right|_{\mathbf{\Delta}=\mathbf{0}} \mathbf{\Delta} = \mathbf{K_D}\,\mathbf{\Delta} \quad (7)$$

In (7), the $N_u \times N_u$ square matrix of the first derivative of $\mathbf{U}$ is the Fisher information matrix, if not some multiplicative terms related to $\frac{E_s}{N_0}$ and the loop bandwidth. From that point of view, the off-diagonal elements appear as describing the coupling due to MAI. Since these are null, there is no coupling. This is a benefit of the DA estimation process.

Splitting u_u^m into U_u and ν_u^m and using (7), the working equation of the linearised closed loop becomes a set of N_u equations of the type

$$\begin{aligned} \hat{\phi}_u^{m+1} &= \hat{\phi}_u^m + K_uF_{u,u}(z)\left(\phi_u - \hat{\phi}_u^m\right) \\ &\quad + K_{0,u}F_{u,u}(z)\nu_u^m \end{aligned} \quad (8)$$

where $K_u = K_{D,u}K_{0,u}$ is the loop gain. Relation (8) is the equation of the loop shown at Figure 1. It illustrates the decoupling between phase recovery processes thanks to the knowledge of interfering users'symbol sequences (DA context). Equation (8) also allows to derive the phase jitter variance $\sigma^2_{\hat{\phi}_u}$ as the variance of the loop noise ν_u^m filtered by the closed loop [10].

$$\sigma^2_{\hat{\phi}_u} = \frac{T}{\left(U_u|_{\Delta=0}\right)^2}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} S_{\hat{\phi}_u}\left(e^{j\omega T}\right)d\omega \quad (9)$$

where $S_{\hat{\phi}_u}(z)$ is the spectral density of the filtered loop noise. The spectral density of the loop noise $S_{\nu_u}(z)$ is the z-transform of the correlation function $C_{u,u}^m$, $S_{\nu_u}(z) = \sum_{m=-\infty}^{+\infty} C_{u,u}^m z^{-m}$ where the correlation $C_{u,u}^m$ writes

$C_{u,u}^m$

$$= \delta(m) \left\{ \begin{array}{l} \sum\limits_{p=-\infty}^{+\infty} \left[\Im \left(x_{u,u}^{p} \right) \right]^2 + \frac{N_0 x_{u,u}^0}{2E_u T} \\ + \sum\limits_{\substack{k=1 \\ k \neq u}}^{N_u} \frac{E_k}{E_u} \sum\limits_{p=-\infty}^{+\infty} \left[\Im \left(e^{j\delta_{k,u}} x_{u,k}^{p} \right) \right]^2 \\ - \sum\limits_{\substack{k=1 \\ k \neq u}}^{N_u} \frac{E_k}{E_u} \sum\limits_{p=-\infty}^{+\infty} \left[\Im \left(e^{j\delta_{k,u}} x_{u,k}^{p} \right) \right]^2 \end{array} \right\} - \left[\Im \left(x_{u,u}^{m} \right) \right]^2 \quad (10)$$

Third and fourth terms of (10) are identical, but their cancellation is only obtained in the case of the multiuser estimator. Missing the fourth term, the single-user estimator cannot compensate the effect of MAI present in (3).

4.2.1 Computational results

Figure 2 illustrates the influence of the correlation properties of the codes and of the load of the system in a situation where the MAI is the only interference (AWGN channel). With orthogonal Hadamard codes, there is no MAI, so the variance of both single-user and multiuser estimators are equal to CRLB. Moving to quasi-orthogonal Gold codes, the single-user curve exhibits an irreducible variance floor. This floor rises along with the load.

Considering a 2-user system over an AWGN channel, the variance curves of both single-user and multiuser estimators are drawn in Figure 3 for different values of the Near-Far ratio. The performance of the single-user estimator being degraded by the MAI, it is not surprising to see that the irreducible variance floor rises as the Near-Far ratio grows.

Finally, the effect of the frequency selectivity of the channel is illustrated in Figure 4. The variances have been computed for two different baud rates in an Hilly Terrain (HT) channel. For both single-user and multiuser estimators, the lower the baud rate, the longer the symbol, and thus the lower the incidence of the ISI. The variance of the multiuser estimator is lower than the one of the single-user because the latter also suffers from MAI. Indeed, neither the single-user nor the multiuser estimator have been designed to face ISI. Inspection of (4) reveals that the ISI influence vanishes when taking the first derivative of the log-likelihood function with respect to the phase parameter. An estimator taking into account ISI is presented in [11].

5 Conclusions

In this paper, a DA phase recovery loop was designed to track the phase in a DS-CDMA multiuser context. The effect of MAI was presented, and a correction term was introduced into a so-called multiuser estimator to cancel its influence. The phase jitter variance was analytically derived, and computed for BPSK-modulated data symbols, considering as well AWGN as dispersive channels. It was shown that the multiuser variance reached Cramér-Rao bound as long as MAI was the only interference in the system, even in Near-Far scenario. However, both single-user and multiuser estimators were affected by ISI, since they were not designed to cancel it.

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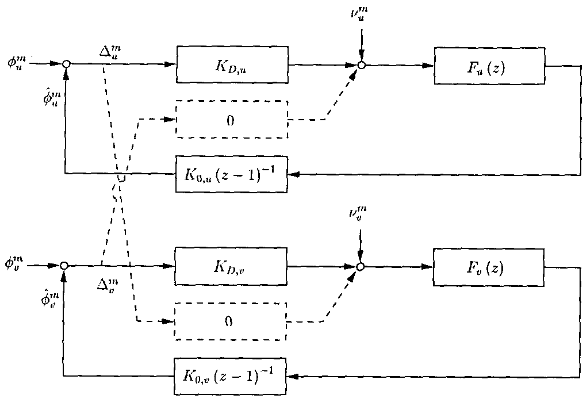

Figure 1: Linearised DA phase recovery loop

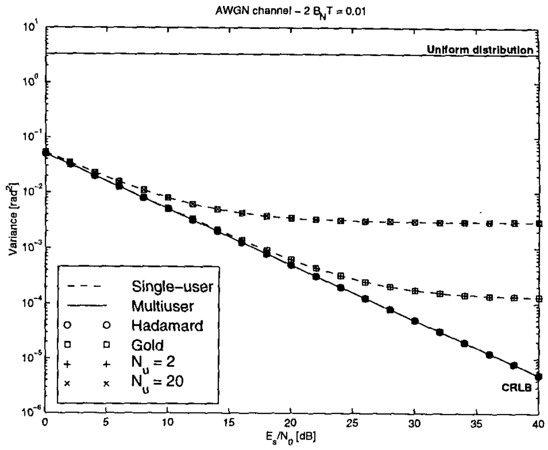

Figure 2: Variance in AWGN channel

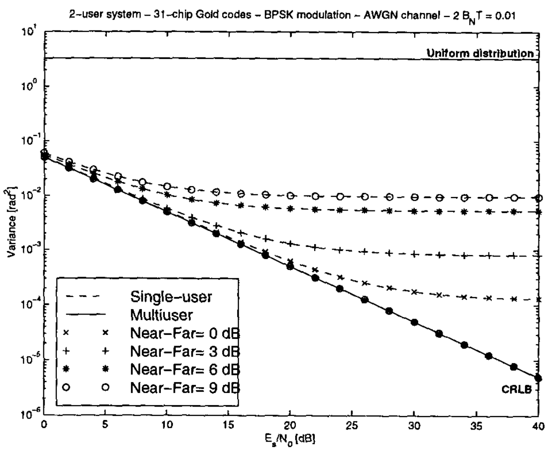

Figure 3: Near-Far effect

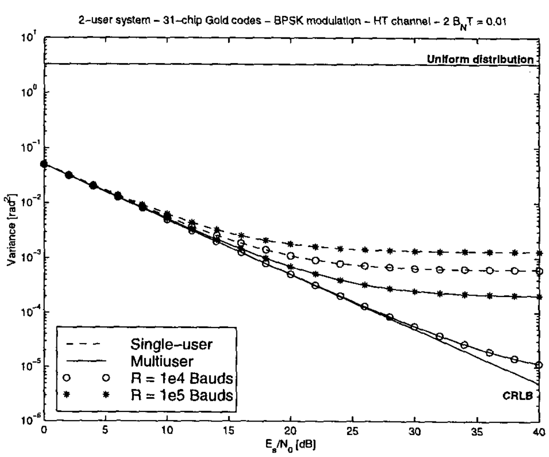

Figure 4: Variance in dispersive channels

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