Open Loop Analysis of Maximum Likelihood Decision-Directed Phase Estimation in CDMA Communication Systems with QPSK Modulation

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Abstract : In CDMA communication systems, the received signal is the sum of contributions from several active users. Instead of dealing with these users separately, in a parallel way, this paper studies multiuser Maximum-Likelihood (ML) phase estimation for a Decision-Directed (DD), QPSK-modulated scenario. Multiuser phase estimation is first defined by means of the ML method. Then, in a 2-user case, open loop analysis is performed and the S-surface is obtained. Finally, the sensitivity of the multiuser phase estimation in different situations is investigated.

1 Introduction

Synchronization issues are the subject of numerous contributions. Analog aspects are summarized in [1] while digital implementation of synchronization devices is treated in [2]. In [2], and in related works [3], multiuser estimation is already introduced. But it is then applied to estimate several parameters (e.g. phase, timing) in a single version of the signal transmitted by a single user.

With the emergence of multiple access techniques such as CDMA [4], the interest of performing simultaneously estimation and detection [5] at the same time over all users has raised. In CDMA communication systems, several users are active on the same fringe of the spectrum at the same time. Thus, the received signal results from the sum of all the contributions from the active users. A conventional way to deal with this situation would be to process the received signal through parallel devices, each one of them being dedicated to a specific user. In such a situation, the coupling between users results in Multiple Access Interference (MAI).

On the other hand, implementations of algorithms aimed at realizing estimation and detection over the whole received signal, have already been proposed, based, for instance, on MUSIC [6] or Expectation-Maximization (EM) [7, 8]. These techniques are all rooted in the Maximum-Likelihood (ML) method [9] but develop suboptimal approaches to circumvent the computational complexity of the practical implementation.

This paper studies multiuser Maximum-Likelihood (ML) Decision-Directed (DD) phase estimation. First of all, section 2 introduces the communication system under study. In the next section, the multiuser phase estimation is derived for a tracking loop implementation. Open loop analysis of this estimation process is performed in section 4, leading to the drawing of S-surfaces. The influence of several parameters on these surfaces is finally investigated.

2 System description

Consider the uplink of a symbol-synchronous coherent CDMA communication system accommodating N_u users. The low-pass equiva-

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lent signal $x_k(t)$ emitted by user k writes:

$$x_k(t) = \sqrt{2P_k} \sum_{n=-\infty}^{\infty} I_k^n s_k(t - nT) \quad (1)$$

where P_k is the emitted power of user k, I_k^n are the QPSK-modulated symbols, with $\sqrt{2}I_k^n = \pm 1 \pm j$, T is the symbol duration and $s_k(t)$ is the spreading waveform for user k

$$s_k(t) = \sum_{n=0}^{N_c-1} a_k^n u(t - nT_c) \quad (2)$$

where $\{a_k^n\}$ is the pseudo-random spreading code, T_c is the chip duration, $N_c = \frac{T}{T_c}$ is the number of chips per symbol and $u(t)$ is a rectangular pulse of duration T_c.

Signals are transmitted through channels having impulse responses $c_k(t)$. Defining

$$h_k(t) = s_k(t) \otimes c_k(t) \quad (3)$$

the signal received at the Base Station (BS), sum of the contributions of the N_u active users, can be written as

$$r(t) = \sum_{k=1}^{N_u} \sqrt{2P_k} e^{j\phi_k} \sum_{n=-\infty}^{\infty} I_k^n h_k(t - nT) + n(t) \quad (4)$$

where ϕ_k is the carrier phase difference and $n(t)$ is the additive white Gaussian noise (AWGN) with one-sided power spectral density N_0. In the following, channel responses $h_k(t)$ will be supposed to be known.

The parameters $\{\phi_k\}$ to be estimated are regarded as deterministic but unknown. This leads to Maximum-Likelihood (ML) estimation [9].

3 DD ML phase estimation in a multiuser context

3.1 ML estimation

ML estimators are theoretically derived from the maximization of the likelihood function. A necessary but not sufficient condition for this maximum can be obtained by differentiating $\Lambda(r)$ with respect to the unknown parameters $\{\phi_k\}$ and setting the result equal to zero [9].

3.2 Multiuser DD ML phase estimation

The multiuser estimation process takes into account the fact that the received signal results from the contribution of several active users. In the assumed multiuser DD context, the logarithm of the likelihood function $\Lambda_L(r)$ writes

$$\begin{aligned}\Lambda_L(r) = & \, Cst - \frac{1}{N_0}\int_{-\infty}^{\infty} |r(t)|^2 dt \\ & + \frac{2T}{N_0} \Re\left[\sum_{k=1}^{N_u} 2P_k e^{j\phi_k} \sum_{n=-\infty}^{\infty} (\hat{I}_k^n)^\star y_k^n\right] \\ & - \frac{2T}{N_0}\sum_{k=1}^{N_u} P_k \sum_{n=-\infty}^{\infty} |\hat{I}_k^n|^2 x_{k,k}^0 \\ & - \frac{2T}{N_0}\sum_{k=1}^{N_u} P_k \sum_{n=-\infty}^{\infty} \sum_{\substack{m=-\infty \\ m\neq n}}^{\infty} \hat{I}_k^n (\hat{I}_k^m)^\star x_{k,k}^{n-m} \\ & - \frac{2T}{N_0}\sum_{k=1}^{N_u} \sum_{\substack{l=1 \\ l\neq k}}^{N_u} \sqrt{P_k P_l}\, e^{j(\phi_l - \phi_k)} \\ & \qquad \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \hat{I}_k^n (\hat{I}_l^m)^\star x_{k,l}^{n-m} \end{aligned} \quad (5)$$

where y_k^n represents the normalized matched filter output and $x_{k,l}^{n-m}$ the normalized channel correlation coefficients

$$y_k^n = \frac{1}{\sqrt{2P_k}T}\int_{-\infty}^{\infty} r(t) h_k^\star(t - nT) dt \quad (6)$$

$$x_{k,l}^{n-m} = \frac{1}{T}\int_{-\infty}^{\infty} h_k(t - nT) h_l^\star(t - mT) dt \quad (7)$$

The last two terms of the expression (5) may be interpreted as interference. The former represents the self-interference (ISI), while the latter is associated with multiple access interference (MAI).

The derivative of (5) with respect to ϕ_u leads to the implicit equation

$$\Im\left\{\begin{array}{l} 2P_u T e^{j\hat{\phi}_u} \sum_{n=-\infty}^{\infty} (\hat{I}_u^n)^\star y_u^n \\ -2T\sqrt{P_u} \sum_{\substack{k=1 \\ k\neq u}}^{N_u} \sqrt{P_k} e^{j(\hat{\phi}_u - \hat{\phi}_k)} \\ \qquad \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} (\hat{I}_u^n)^\star \hat{I}_k^m x_{k,u}^{m-n} \end{array}\right\} = 0 \quad (8)$$

where index k points interfering contributions with respect to user u. The left side of (8) can be used as an error signal in a tracking loop device like a Phase-Locked Loop (PLL) whose equation would be

$$\hat{\phi}_u^{(n+1)} = \hat{\phi}_u^n + K_u u_{\phi_u}(n) \tag{9}$$

3.3 2-user DD ML phase estimation

In the following, the multiuser context will be restricted to a 2-user case. Moreover, single-ray channels presenting a fading β_i will be used. In such a scenario, the error signal sample derived from ML estimation for user 1 in the presence of interfering user 2 writes

$$u_{\phi_1}(n) = \Im \left[\begin{array}{l} 2\,P_1\,\beta_1^2\,e^{j(\phi_1^0-\hat{\phi}_1)} \\ \quad (\hat{I}_1^n)^\star \sum\limits_{m=-\infty}^{\infty} I_1^m x_{11}^{n-m} \\ +2\sqrt{P_1 P_2}\,\beta_1\beta_2\,e^{j(\phi_2^0-\hat{\phi}_1)} \\ \quad (\hat{I}_1^n)^\star \sum\limits_{m=-\infty}^{\infty} I_2^m x_{12}^{n-m} \\ -2\sqrt{P_1 P_2}\,\beta_1\beta_2\,e^{j(\hat{\phi}_2-\hat{\phi}_1)} \\ \quad \hat{I}_2^n \sum\limits_{m=-\infty}^{\infty} (\hat{I}_1^m)^\star x_{12}^{n-m} \\ +\frac{\sqrt{2}}{T}\sqrt{P_1}\,\beta_1\,e^{-j\hat{\phi}_1} \\ \quad (\hat{I}_1^n)^\star \int_{-\infty}^{\infty} n(t) h_1^\star(t-nT)\,dt \end{array} \right] \tag{10}$$

4 Open Loop Analysis

4.1 Analytical expression of the average ML error signal

Defining

$$\begin{aligned} I_k^n &= a_k^n + j b_k^n && (11)\\ \hat{a}_k^n &= \frac{\sqrt{2}}{2} Sgn\left[\Re\left(y_k^n\right)\right] && (12)\\ \hat{b}_k^n &= \frac{\sqrt{2}}{2} Sgn\left[\Im\left(y_k^n\right)\right] && (13)\\ n(t) &= n_c(t) + j n_s(t) && (14)\\ \Delta_i &= \phi_i^0 - \hat{\phi}_i && (15) \end{aligned}$$

the open loop analysis can be performed. The mathematical average of (10) conditioned on $\{\phi_i, i=1,2\}$ is written

$$U_{\phi_1} = E\left[u_{\phi_1(n)} \middle| \phi_i = \Delta_i, \hat{\phi}_i = 0 \right] \quad i = 1,2$$

$$\begin{aligned} &= \Im\left\{ \begin{array}{l} 2\,P_1\,\beta_1^2\,e^{j\Delta_1} \\ \sum\limits_{m=-\infty}^{\infty} \left[\begin{array}{l} E(\hat{a}_1^n a_1^m) \\ +E(\hat{b}_1^n b_1^m) \end{array} \right] x_{11}^{n-m} \end{array} \right\} \\ &+\Re\left\{ \begin{array}{l} 2\,P_1\,\beta_1^2\,e^{j\Delta_1} \\ \sum\limits_{m=-\infty}^{\infty} \left[\begin{array}{l} E(\hat{a}_1^n b_1^m) \\ +E(\hat{b}_1^n a_1^m) \end{array} \right] x_{11}^{n-m} \end{array} \right\} \\ &+\Im\left\{ \begin{array}{l} 2\sqrt{P_1 P_2}\,\beta_1\beta_2\,e^{j\Delta_2} \\ \sum\limits_{m=-\infty}^{\infty} \left[\begin{array}{l} E(\hat{a}_1^n a_2^m) \\ +E(\hat{b}_1^n b_2^m) \end{array} \right] x_{12}^{n-m} \end{array} \right\} \\ &+\Re\left\{ \begin{array}{l} 2\sqrt{P_1 P_2}\,\beta_1\beta_2\,e^{j\Delta_2} \\ \sum\limits_{m=-\infty}^{\infty} \left[\begin{array}{l} E(\hat{a}_1^n b_2^m) \\ +E(\hat{b}_1^n a_2^m) \end{array} \right] x_{12}^{n-m} \end{array} \right\} \\ &-\Im\left\{ \begin{array}{l} 2\sqrt{P_1 P_2}\,\beta_1\beta_2 \\ \sum\limits_{m=-\infty}^{\infty} \left[\begin{array}{l} E(\hat{a}_1^n \hat{a}_2^m) \\ +E(\hat{b}_1^n \hat{b}_2^m) \end{array} \right] x_{12}^{n-m} \end{array} \right\} \\ &-\Re\left\{ \begin{array}{l} 2\sqrt{P_1 P_2}\,\beta_1\beta_2 \\ \sum\limits_{m=-\infty}^{\infty} \left[\begin{array}{l} E(\hat{a}_1^n \hat{b}_2^m) \\ +E(\hat{b}_1^n \hat{a}_2^m) \end{array} \right] x_{12}^{n-m} \end{array} \right\} \\ &+\tfrac{\sqrt{2}}{T}\beta_1 P_1 \left\{ E\left[\hat{a}_1^n \int_{-\infty}^{\infty} n_s(t) h_1^\star(t-nT)\,dt \right] \right\} \\ &-\tfrac{\sqrt{2}}{T}\beta_1 P_1 \left\{ E\left[\hat{b}_1^n \int_{-\infty}^{\infty} n_c(t) h_1^\star(t-nT)\,dt \right] \right\} \\ &-\tfrac{\sqrt{2}}{T}\beta_1 P_1 \left\{ E\left[\hat{a}_1^n \int_{-\infty}^{\infty} n_c(t) h_1^\star(t-nT)\,dt \right] \right\} \\ &-\tfrac{\sqrt{2}}{T}\beta_1 P_1 \left\{ E\left[\hat{b}_1^n \int_{-\infty}^{\infty} n_s(t) h_1^\star(t-nT)\,dt \right] \right\} \end{aligned} \tag{16}$$

Due to the QPSK signal constellation symmetry, the noise contribution in (16) disappears. The average error signal in the multiuser context contains three main contributions. The two first contributions come from the expansion of the matched filter output. The last one is a specific contribution introduced due to the multiuser estimation studied. The detailed expressions of the first order statistics in (16) are presented in Appendix A.

The average error expression simplifies if data flows from the two users are regarded as statistically independent. Then, only contributions with $n = m$ are left in (16). This also reduces the specification of the coupling between these users x_{12}^{n-m} to the only value x_{12}^0.

4.2 Computation of the average ML error signal

Expression (16) has been computed with Matlab, considering different values of its parameters ($\frac{E_b}{N_0}$, coupling factor x_{12}^0, phase offset $(\phi_2^0 - \phi_1^0)$ and channel fadings $\beta_i, i = 1,2$). Results are shown in the following figures as a function of Δ_1 and/or Δ_2.

In figure 1, the S-surface, that is the error signal as a function of both Δ_1 and Δ_2 is drawn with $x_{12}^0 = 0$. Since there is no coupling between the two users, the S-surface of the multiuser estimation process reduces to the S-curve of a single user process. One can notice the $\frac{\pi}{2}$ periodicity with respect to Δ_1 while the error does not depend on Δ_2. On the right side of figure 1, the projection of the S-surface on the Δ_1-plane is also presented, to obtain the typical S-curve of single user QPSK reception [10],[11]. When coupling is introduced in the system (figures 2 and 3), a dependency of the multiuser estimation to Δ_2 appears which exhibit the same $\frac{\pi}{2}$ periodicity. There is no such dependency on single user estimation S-surfaces, which keep on exhibitting the same S-surface as in figure 1.

In the present case, an interesting value to monitor on these surfaces is the value of user 1 error detector slope at the stable tracking point ($\Delta_1 = 0$), since this value usually helps to design an equivalent linearized model of the tracking loop.

The influence on the coupling factor x_{12}^0 on this slope is presented in figure 4 for two values of $\frac{E_b}{N_0}$ and considering $(\phi_2^0 - \phi_1^0) = 0$ and $\beta_1 = \beta_2$. The greater the coupling is, the better the multiuser estimation performs with respect to single user estimation at $\Delta_2 = 0$ (figure 4a). On the other hand, the performance degradation is worse with growing coupling factors when Δ_2 is away from its stable point (figure 4b).

Figure 5 presents the value of the error detector slope at $\Delta_1 = 0$ as a function of Δ_2 for several values of $\frac{E_b}{N_0}$, $(\phi_2^0 - \phi_1^0)$ and β_1, keeping $x_{12}^0 = 0.2$ and $\beta_2 = 1$. All curves come by pair. Since it does not depend on Δ_2, the curve of the single user estimation process is horizontal. This line is drawn at the value reached by a single user estimation device. The other curve in the same style represents the error detector slope of a multiuser estimation process in the same situation. Moreover, a small cross ($\times$) is placed on the horizontal line at the value of the offset $(\phi_2^0 - \phi_1^0)$.

As shown in [10],[11] and on figure 5, when $\frac{E_b}{N_0}$ grows from 5 to 15 dB, the values of the slope raise, moving closer to 1.

The influence of the offset $(\phi_2^0 - \phi_1^0)$ is more subtle. While offset does not seem to influence the value of the error detector slope when there is no fading of user 1, the effect of the offset appears when reducing the value of β_1 with respect to β_2, so to create a near-far effect. Then, one can notice (figures 5d and 5f) that the slopes obtained with the multiuser estimation process are greater than the single ones as long as the offset $(\phi_2^0 - \phi_1^0)$ is not higher than 20^o. Indeed, for offset values lower than or equal to 20^o, the multiuser slope is above the single user one, and present a bias towards the offset value. But for offset values higher than 20^o, the multiuser slope is no longer the steepest one.

5 Conclusion

This paper investigated multiuser Maximum-Likelihood Decision-Directed phase estimation. After deriving the analytical expression of the error signal driving a tracking loop in a 2-user case, the open loop analysis of this tracking device was performed and lead to the drawing of S-surfaces. These surfaces resulting from single user and multiuser estimation processes were compared and the influence of several parameters on the value of the error detector slope was studied.

A First Order Statistics

The purpose of this appendix is to present the expressions of the First Order Statistics in a multiuser QPSK Decision-Directed context. This presentation will be limited to Data $\times$ Data first order statistics, since these are the only statistics used in this paper. Defining the

filtered noise contributions as

$$\mu_k^n = \Re\left[\frac{1}{\sqrt{2P_kT}} \int_{-\infty}^{\infty} n(t)\ h_k^\star(t-nT)\ dt\right] \quad (17)$$

$$\nu_k^n = \Im\left[\frac{1}{\sqrt{2P_kT}} \int_{-\infty}^{\infty} n(t)\ h_k^\star(t-nT)\ dt\right] \quad (18)$$

and, for $k = 1..8$, the following X_k and Y_k

$X_k =$

$$\beta_1\, x_{11}^0 \cos\left[\frac{\pi}{4} + (-1)^{\lfloor\frac{k-1}{4}\rfloor}\Delta_1\right] + (-1)^{\lfloor\frac{k-1}{2}\rfloor}\sqrt{\frac{P_2}{P_1}}\,\beta_2\, x_{12}^0 \cos\left\{\frac{\pi}{4} + (-1)^{k-1}\left[\Delta_1 + (\phi_2^0 - \phi_1^0)\right]\right\} \quad (19)$$

$$Y_k = \sqrt{\frac{P_1}{P_2}}\,\beta_1\, x_{21}^0 \cos\left\{\frac{\pi}{4} + (-1)^{k-1}\left[\Delta_2 - (\phi_2^0 - \phi_1^0)\right]\right\} + (-1)^{\lfloor\frac{k-1}{2}\rfloor}\beta_2\, x_{22}^0 \cos\left[\frac{\pi}{4} + (-1)^{k-1}\Delta_2\right] \quad (20)$$

the analytical expressions of the statistics are written as below. The statistical independence between the data flows of the users drive most of these statistics to zero, mainly these for which temporal indexes n and m differ $(n - m \neq 0)$. As a result, only terms with $n = m$ are non zero.

$$E(\hat{a}_1^n a_1^m)|_{m=n} = \frac{1}{16}\sum_{k=1}^{8}\left[1 - 2P\left(\mu_1^n \geq X_k\right)\right] \quad (21)$$

$$E(\hat{b}_1^n b_1^m)\Big|_{m=n} = \frac{1}{16}\sum_{k=1}^{8}\left[1 - 2P\left(\nu_1^n \geq X_k\right)\right] \quad (22)$$

$$E(\hat{a}_1^n b_1^m)|_{m=n} = \frac{1}{8}\sum_{k=1}^{4}\begin{bmatrix} P\left(\mu_1^n \geq X_{(k+4)}\right) \\ -P\left(\mu_1^n \geq X_k\right)\end{bmatrix} \quad (23)$$

$$E(\hat{b}_1^n a_1^m)\Big|_{m=n} = \frac{1}{8}\sum_{k=1}^{4}\begin{bmatrix} P\left(\nu_1^n \geq X_k\right) \\ -P\left(\nu_1^n \geq X_{(k+4)}\right)\end{bmatrix} \quad (24)$$

$$E(\hat{a}_1^n a_2^m)|_{m=n} = \frac{1}{8}\sum_{k=1}^{4}\begin{bmatrix} P\left(\mu_1^n \geq X_{k+2\lceil\frac{k-1}{2}\rceil}\right) \\ -P\left(\mu_1^n \geq X_{(k+2\lfloor\frac{k-1}{2}\rfloor)}\right)\end{bmatrix} \quad (25)$$

$$E(\hat{b}_1^n b_2^m)\Big|_{m=n} = \frac{1}{8}\sum_{k=1}^{4}\begin{bmatrix} P\left(\nu_1^n \geq X_{k+2\lceil\frac{k-1}{2}\rceil}\right) \\ -P\left(\nu_1^n \geq X_{(k+2\lfloor\frac{k-1}{2}\rfloor)}\right)\end{bmatrix} \quad (26)$$

$$E(\hat{a}_1^n b_2^m)|_{m=n} = \frac{1}{8}\sum_{k=1}^{4}\begin{bmatrix} P\left(\mu_1^n \geq X_{k+2\lfloor\frac{k-1}{2}\rfloor-1}\right) \\ -P\left(\mu_1^n \geq X_{3\lceil\frac{k-1}{2}\rceil+2(-1)^i}\right)\end{bmatrix} \quad (27)$$

$$E(\hat{b}_1^n a_2^m)\Big|_{m=n} = \frac{1}{8}\sum_{k=1}^{4}\begin{bmatrix} P\left(\nu_1^n \geq X_{3\lceil\frac{k-1}{2}\rceil+2(-1)^i}\right) \\ -P\left(\nu_1^n \geq X_{k+2\lfloor\frac{k-1}{2}\rfloor-1}\right)\end{bmatrix} \quad (28)$$

$$E(\hat{a}_1^n \hat{a}_2^m)|_{m=n} = \frac{1}{16}\sum_{k=1}^{8}\begin{bmatrix} 4P\left(\mu_1^n \geq X_k\right) \\ P\left(\mu_2^n \geq Y_k\right) \\ -2P\left(\mu_1^n \geq X_k\right) \\ -2P\left(\mu_2^n \geq Y_k\right) + 1\end{bmatrix} \quad (29)$$

$$E(\hat{b}_1^n \hat{b}_2^m)\Big|_{m=n} = \frac{1}{16}\sum_{k=1}^{8}\begin{bmatrix} 4P\left(\nu_1^n \geq X_k\right) \\ P\left(\nu_2^n \geq Y_k\right) \\ -2P\left(\nu_1^n \geq X_k\right) \\ -2P\left(\nu_2^n \geq Y_k\right) + 1\end{bmatrix} \quad (30)$$

$$E(\hat{a}_1^n \hat{b}_2^m)\Big|_{m=n} = \frac{1}{16}\sum_{k=1}^{4}\begin{bmatrix} \begin{pmatrix} 4P\left(\mu_1^n \geq X_k\right) \\ P\left(\nu_2^n \geq Y_{5+k'}\right) \\ -2P\left(\mu_1^n \geq X_k\right) \\ -2P\left(\nu_2^n \geq Y_{5+k'}\right) \\ +1 \end{pmatrix} \\ -\begin{pmatrix} 4P\left(\mu_1^n \geq X_{5+k'}\right) \\ P\left(\nu_2^n \geq Y_k\right) \\ -2P\left(\mu_1^n \geq X_{5+k'}\right) \\ -2P\left(\nu_2^n \geq Y_k\right) \\ +1 \end{pmatrix}\end{bmatrix} \quad (31)$$

$$E(\hat{b}_1^n \hat{a}_2^m)\Big|_{m=n} = \frac{1}{16}\sum_{k=1}^{4}\begin{bmatrix} \begin{pmatrix} 4P\left(\mu_2^n \geq Y_k\right) \\ P\left(\nu_1^n \geq X_{5+k'}\right) \\ -2P\left(\mu_2^n \geq Y_k\right) \\ -2P\left(\nu_1^n \geq X_{5+k'}\right) \\ +1 \end{pmatrix} \\ -\begin{pmatrix} 4P\left(\mu_2^n \geq Y_{5+k'}\right) \\ P\left(\nu_1^n \geq X_k\right) \\ -2P\left(\mu_2^n \geq Y_{5+k'}\right) \\ -2P\left(\nu_1^n \geq X_k\right) \\ +1 \end{pmatrix}\end{bmatrix} \quad (32)$$

where k' is the rest of the division of k by 4, $\lceil k \rceil$ represents the least integral value strictly greater than k and $\lfloor k \rfloor$ stands for the greatest integer value lower than or equal to k.

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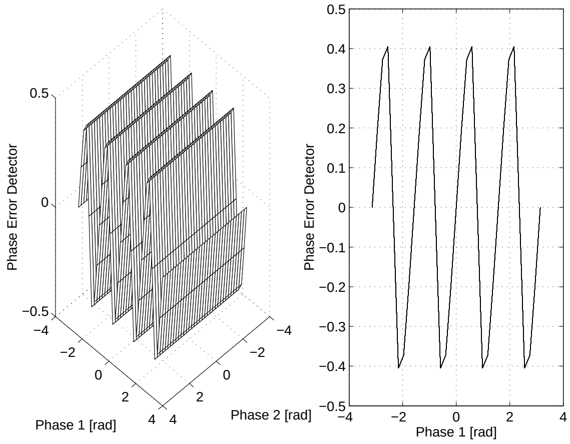


Figure 1: Multiuser ML estimation S-surface, no coupling ($x_{12}^0 = 0$)

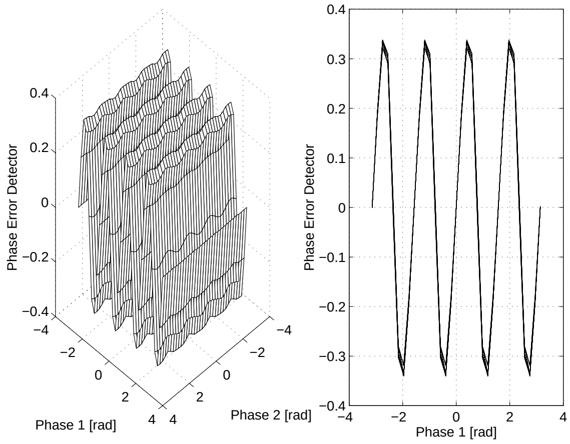


Figure 2: Multiuser ML estimation S-surface, $x_{12}^0 = 0.2$, $(\phi_2^0 - \phi_1^0) = 10^o$ and $\beta_1 = \beta_2$

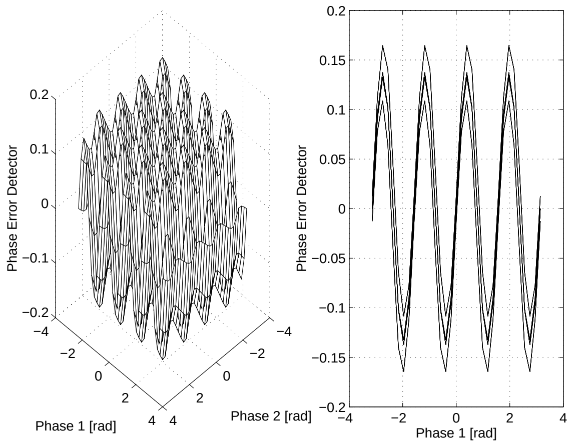
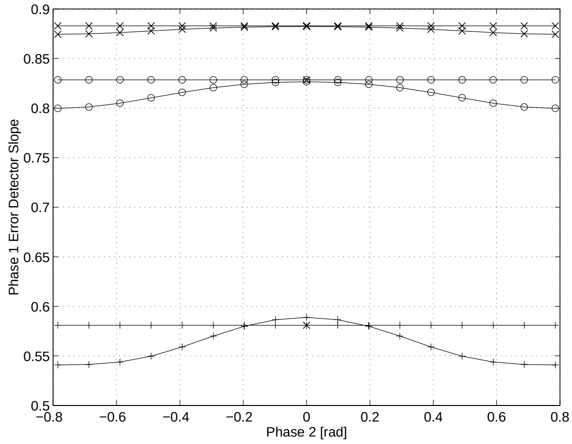
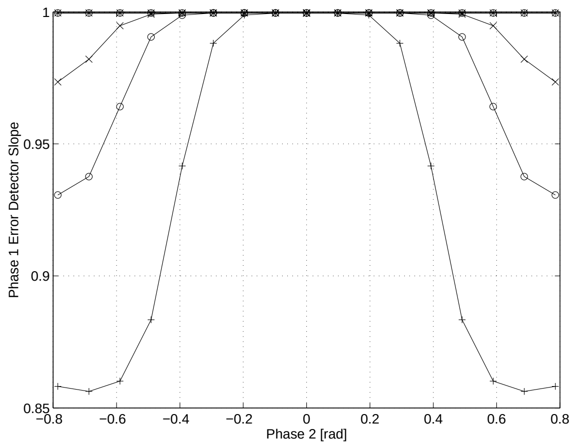


Figure 3: Multiuser ML estimation S-surface, $x_{12}^0 = 0.2$, $(\phi_2^0 - \phi_1^0) = 10^o$ and $\beta_1 = 0.5\,\beta_2$

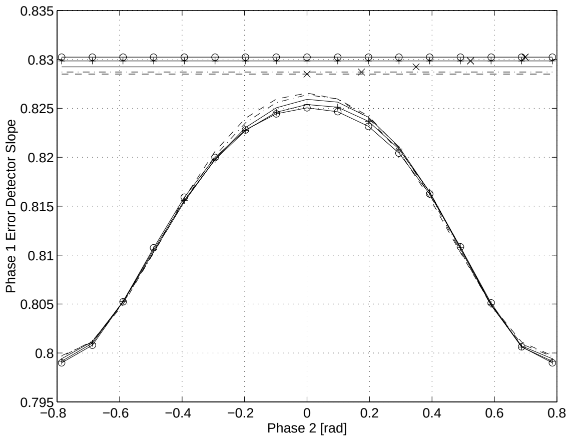


(a) $\frac{E_b}{N_0} = 5\ dB$, $\beta_1 = \beta_2$, $(\phi_2^0 - \phi_1^0) = 0^o$

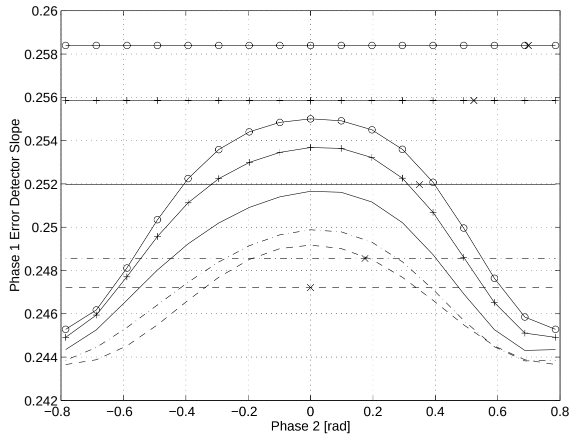


(b) $\frac{E_b}{N_0} = 15\ dB$, $\beta_1 = \beta_2$, $(\phi_2^0 - \phi_1^0) = 0^o$

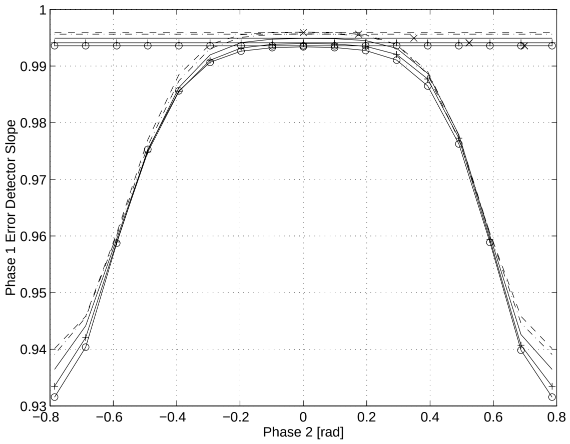
Figure 4: Error Detector Slopes as a function of Δ_2, with coupling x_{12}^0 used as a parameter taking values 0.1 (continuous line with crosses), 0.2 (continuous line with circles) and 0.4 (continuous line with +)



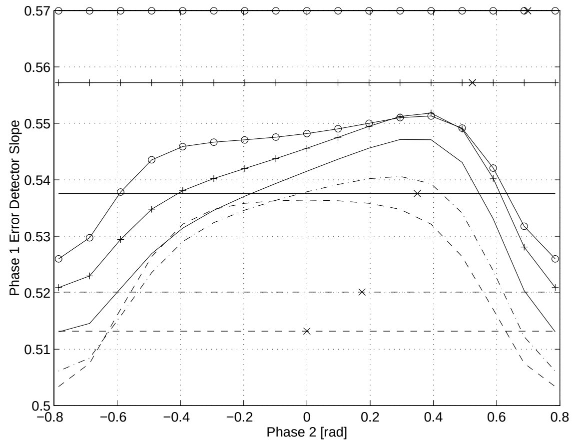
(a) $\frac{E_b}{N_0} = 5\ dB$, $\beta_1 = \beta_2$, $x_{12}^0 = 0.2$



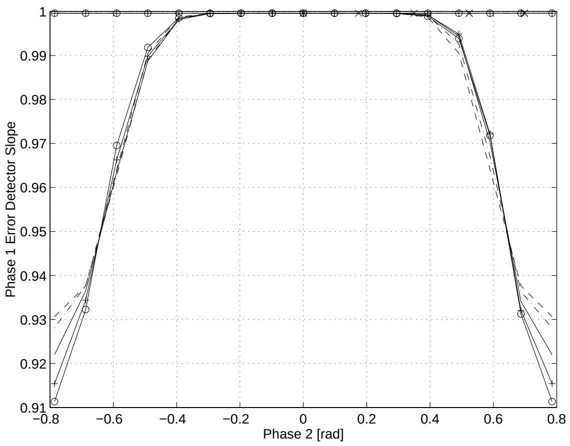
(b) $\frac{E_b}{N_0} = 5\ dB$, $\beta_1 = 0.5\ \beta_2$, $x_{12}^0 = 0.2$



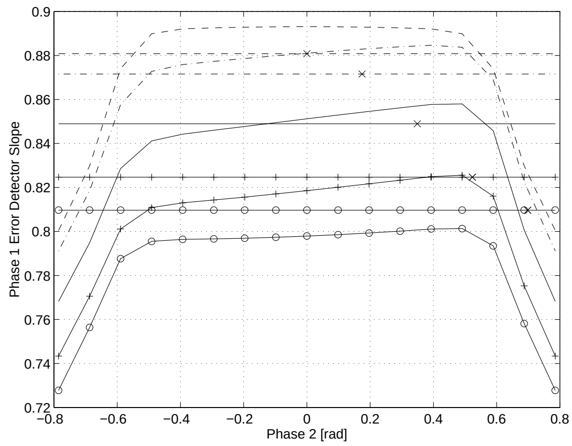
(c) $\frac{E_b}{N_0} = 10\ dB$, $\beta_1 = \beta_2$, $x_{12}^0 = 0.2$



(d) $\frac{E_b}{N_0} = 10\ dB$, $\beta_1 = 0.5\ \beta_2$, $x_{12}^0 = 0.2$



(e) $\frac{E_b}{N_0} = 15\ dB$, $\beta_1 = \beta_2$, $x_{12}^0 = 0.2$



(f) $\frac{E_b}{N_0} = 15\ dB$, $\beta_1 = 0.5\ \beta_2$, $x_{12}^0 = 0.2$

Figure 5: Δ_1 Error Detector Slopes as a function of Δ_2, with offset $(\phi_2^0 - \phi_1^0)$ used as a parameter taking values 0^o (dashed line), 10^o (dashed-dotted line), 20^o (continuous line), 30^o (continuous line with +) and 40^o (continuous line with circles)