Maximum Likelihood Joint Phase Estimators in CDMA Communication Systems

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Abstract

In this paper, the problem of carrier phase estimation in a Multi User CDMA context is considered. Regarding the phases as deterministic but unknown parameters, maximum-likelihood (ML) estimation is applied in both data-aided (DA) and non-data aided (NDA) contexts, leading to analytical expressions of ML phase estimators. Variances of these estimators are estimated in a BPSK modulation scheme and compared with respect to various benchmarks.

1 Introduction

Efficient digital communications require the determination of various parameters characterizing the transmission link : timing, channel responses, signal power,... and phase for coherent detection. Moreover, in a CDMA system, parameter estimation is complicated by the interference due to other active users. Interference between users, or Multiple Access Interference (MAI), results in biased parameters and/or important jitter.

This paper presents analytical expressions of Maximum-Likelihood joint phase estimators to be used in a CDMA communication system. After a brief system description in Section 2, Data-Aided ML joint phase estimation is studied in Section 3. An estimator is derived by linearizing the ML equation. The quality of this ML DA estimator is estimated by computing its variance and comparing it with respect to the Cramer-Rao bound and the variance of a conventional estimator. In Section 4, a Non-Data Aided joint phase estimator is derived, following a method described in [1]. The variance of this ML NDA estimator is compared to that corresponding to the Cramer-Rao bound.

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2 System description

The lowpass equivalent signal transmitted by user k is given by

$$x_k(t) = \sqrt{2\,P_k}\sum_{n=-\infty}^{\infty} I_k^n a_k(t) u(t-nT) \tag{1}$$

where I_k^n are the data produced by user k, $a_k(t)$ is the waveform associated with the periodical pseudo-noise sequence multiplying the signal for user k and $u(t)$ is the symbol shape.

Assuming that the channel between user k and the receiver is a linear channel with equivalent lowpass impulse response $c_k(t)$, the received signal from N_u active users is given by

$$r(t) = \sum_{k=1}^{N_u}\sqrt{2\,P_k}\;e^{-j\phi_k}\sum_{n=-\infty}^{\infty} I_k^n h_k(t-nT) + n(t) \tag{2}$$

where $h_k(t) = [a_k(t)u(t)] \otimes c_k(t)$, ϕ_k is the phase parameter for user k, and n(t) is the additive white gaussian noise (AWGN) with one-sided power spectral density of N_0.

The parameters to be estimated are regarded as deterministic but unknown. This leads thus to maximum-likelihood estimation [2]. The logarithm of the likelihood function $\Lambda_L(r)$ writes

$$\begin{aligned}\Lambda_L(r) \;=\; & Cst - \frac{1}{N_0}\int_{-\infty}^{\infty}|r(t)|^2 + \frac{2}{N_0}\Re\left[\sum_{k=1}^{N_u} 2\,P_k\;e^{j\phi_k}\sum_{n=-\infty}^{\infty}(I_k^n)^\star y_k^n\right] \\ & -\frac{1}{N_0}\sum_{k=1}^{N_u} 2\,P_k\sum_{n=-\infty}^{\infty}|I_k^n|^2 x_{k,k}^{n,n} - \frac{1}{N_0}\sum_{k=1}^{N_u} 2\,P_k\sum_{n=-\infty}^{\infty}\sum_{\substack{m=-\infty\\ m\neq n}}^{\infty} I_k^n (I_k^m)^\star x_{k,k}^{n,m} \\ & -\frac{2}{N_0}\sum_{k=1}^{N_u}\sum_{\substack{l=1\\ l\neq k}}^{N_u}\sqrt{P_k\;P_l}\;e^{j(\phi_l-\phi_k)}\sum_{n=-\infty}^{\infty}\sum_{\substack{m=-\infty\\ m\neq n}}^{\infty} I_k^n (I_l^m)^\star x_{k,l}^{n,m}\end{aligned} \tag{3}$$

where y_k^n represents the normalized matched filter output and $x_{k,l}^{n,m}$ the normalized channel correlation coefficients

$$y_k^n = \frac{1}{\sqrt{2\,P_k}\;T}\int_{-\infty}^{\infty} r(t) h_k^\star(t-nT)dt \tag{4}$$

$$x_{k,l}^{n,m} = \frac{1}{T}\int_{-\infty}^{\infty} h_k(t-nT) h_l^\star(t-mT)dt \tag{5}$$

The last two terms of the expression (3) hereabove may be interpreted as interference. The former represents the self-interference (ISI), while the latter is associated with multiple access interference (MAI).

3 Data-Aided Joint Phase Estimation Estimation

3.1 Derivation of the estimator

The ML estimators are theoretically derived from the maximization of the likelihood function. A necessary but not sufficient condition for this maximum can be obtained by differentiating $\Lambda(r)$ with respect to the unknown parameters Φ and setting the result equal to zero [2].

$$\left.\frac{\partial\Lambda(r)}{\partial\Phi}\right|_{\Phi=\hat{\Phi}} = 0 \tag{6}$$

Moreover, in the present case, $e^{j\hat{\phi}_u}$ is linearized around the true value of the parameter to be estimated, such that

$$e^{j\hat{\phi}_{u,DA}} = e^{j\phi_u}\left(1 + j(\hat{\phi}_{u,DA} - \phi_u)\right) \tag{7}$$

With such a linearization, the following developments fall under the hypothesis of high SNR. Using (6) and (7), the first derivative of (3) leads to the following condition

$$\begin{aligned}\Im\Bigg[&2\,P_u\,T\,e^{j\phi_u}\left(1 + j(\hat{\phi}_{u,DA} - \phi_u)\right)\sum_{n=-\infty}^{\infty}(I_u^n)^\star y_u^n \\ &-2\,T\sqrt{P_u\,P_k}\,e^{j\phi_u}\left(1 + j(\hat{\phi}_{u,DA} - \phi_u)\right) \\ &\sum_{k=1}^{N_u} e^{-j\phi_k}\left(1 - j(\hat{\phi}_{k,DA} - \phi_k)\right)\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}(I_u^n)^\star I_k^m x_{k,u}^{m,n}\Bigg] = 0\end{aligned} \tag{8}$$

Solving (8) gives

$$(\hat{\phi}_{u,DA} - \phi_u) = -\frac{\Im\,[A_u]}{\Re\,[A_u]} \tag{9}$$

where

$$\begin{aligned}A_u \;=\; &2\,P_u\,T\,e^{j\phi_u}\sum_{n=-\infty}^{\infty}(I_u^n)^\star y_u^n \\ &-2\,T\sqrt{P_u P_k}\sum_{k=1}^{N_u}\left(1 - j(\hat{\phi}_{k,DA} - \phi_k)\right)\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}(I_u^n)^\star I_k^m x_{k,u}^{m,n}\end{aligned} \tag{10}$$

One can note in (9) that the linearization of $e^{j\hat{\phi}_{u,DA}}$ has led to a result similar to classical ones [3], but where the highly nonlinear arctan function has disappeared.

The following developments are limited to the 2-user case. Expanding y_u^n, direct ($x_{u,u}^{n,n}$) and ISI ($x_{u,u}^{m,n}$, $m \neq n$) terms vanish from the numerator as real terms. Moreover, regarding MAI and noise contributions in the denominator as negligible, the ML DA joint phase estimators are of the type

$$(\hat{\phi}_{u,DA} - \phi_u) = \frac{\Im[Noise_u]\,\Re[Direct_v + ISI_v] + \Re[MAI_{v,u}]\,\Im[Noise_v]}{\Re[Direct_u + ISI_u]\,\Re[Direct_v + ISI_v] - \Re[MAI_{v,u}]\,\Re[MAI_{u,v}]} \tag{11}$$

where

$$Direct_u = \sum_{n=-\infty}^{\infty} |I_u^n|^2 \, x_{u,u}^{n,n} \tag{12}$$

$$ISI_u = \sum_{n=-\infty}^{\infty} \sum_{\substack{m=-\infty \\ m\neq n}}^{\infty} (I_u^n)^\star I_u^m x_{u,u}^{m,n} \tag{13}$$

$$MAI_{u,v} = 2\sqrt{\frac{P_u}{P_v}}\, e^{(\phi_v-\phi_u)} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} (I_v^n)^\star I_u^m x_{u,v}^{m,n} \tag{14}$$

$$Noise_u = \sqrt{\frac{2}{P_v}}\, \frac{e^{j\phi_u}}{T} \sum_{n=-\infty}^{\infty} (I_u^n)^\star \int_{-\infty}^{+\infty} n(t) h_u^\star(t-nT)dt \tag{15}$$

3.2 Variance of the estimator - Computational results

To be able to compute the variance of (11), the denominator is replaced by its mathematical expectation. Expression of the variance is then derived in the case of a BPSK modulation. It depends on the structure of the correlation factors $x_{u,v}^{m,n}$, the phase difference between users, and the size of the averaging window.

This expression of the variance has been computed in different scenarii. The results presented here (Fig. 1) are related to a situation where the coupling between users is as high as 20 % of the direct ray, without spreading around it ($x_{u,v}^{n,n} = 0.2\ x_{u,u}^{n,n};\ x_{u,u}^{m,n} \equiv x_{u,v}^{m,n} \equiv 0$ for $m \neq n$). Different phase offsets are tested, and the resistance to the Near-Far effect is also shown. Variance of the ML DA estimators is presented with respect to the Cramer-Rao bound and the variance of a conventional estimator. By conventional estimator, one should understand a process not modelizing the possible coupling, and dealing with the matched filters output y_u^n as if they were corresponding to a single active user, although several are interfering through MAI.

As $\frac{E_b}{N_0}$ grows, ML DA estimators appear (Fig. 1, left column) to be performing better than the conventional ones, excepted when the phase difference reaches zero. These estimators are not limited by the threshold effect visible on the performance of the conventional estimator used in a multiple access system. Asymptotically, their variance tends to the Cramer-Rao bound.

This independence of the treshold effect is especially interesting when estimation has to face a Near-Far effect (Fig. 1, right column. Near-Far power ratio is 1:10). The Near-Far effect increases the value of the treshold limiting the performance of the conventional estimator in the multiple access system. Joint estimation outperforms then such an estimator.

4 Non Data-Aided Joint Phase Estimation

4.1 Derivation of the estimator

The NDA maximum-likelihood estimators result from the maximization of the likelihood function $\Lambda(r)$ averaged over the statistical symbol distribution.

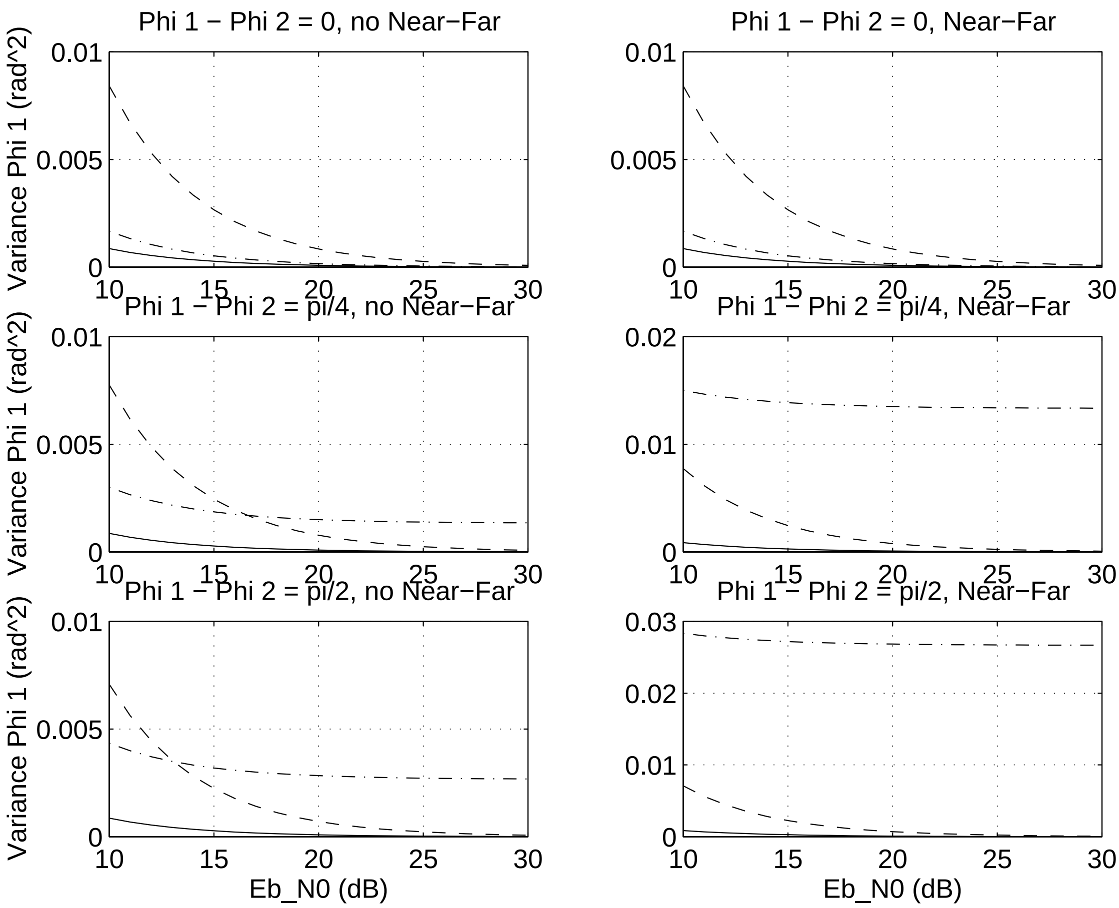

Figure 1: Variance of DA ML estimators (Joint - -, Cramer-Rao —, Conventional - .)

$$\bar{\Lambda}(r) = \int_{-\infty}^{+\infty} \Lambda(r)\; p(I_k^n)\; dI_k^n \qquad (16)$$

In this case, symbols are regarded as independent, identically distributed zero-mean binary data. The averaging of the likelihood function $\Lambda(r)$ over the pdf of I_k^n is hardly analytically manageable, due to the presence of the exponential function. Moeneclaey and de Jonghe proposed in [1] a way to derive the NDA likelihood function $\Lambda(r)$, based on the expansion of the exponential function into a power series. Applying their method, $\bar{\Lambda}(r)$ writes

$$\begin{aligned} \bar{\Lambda}(r) &= \frac{1}{2} \sum_{k=1}^{N_u} \left(\frac{4\; P_k\; T}{N_0} \right)^2 \sum_{n=-\infty}^{+\infty} \Re \left[e^{j\,\phi_k} \left(I_k^n \right)^\star y_k^n \right]^2 \\ &+ \frac{1}{2} \sum_{k=1}^{N_u} \sum_{l=1}^{N_u} \frac{4\; P_k\; P_l}{N_0^2} \sum_{p=-\infty}^{+\infty} \sum_{q=-\infty}^{+\infty} e^{2j(\phi_l - \phi_k)} \left[I_k^p \left(I_l^q \right)^\star x_{k,l}^{p,q} \right]^2 \end{aligned} \qquad (17)$$

To notice is the NDA-typical quadratic operator applying to each term of this expression.

In practice, the maximization of the NDA likelihood function (17) is realized, similarly to the DA case, by founding parameter values zero forcing the first derivative of $\bar{\Lambda}(r)$ with respect to them. Considering BPSK modulation, conditions to satisfy are of the form

$$\Im\left[e^{2j\phi_u}\sum_{n=-\infty}^{+\infty}(y_u^n)^2+e^{2j\phi_u}\sum_{k=1}^{N_u}e^{-2j\phi_k}\sum_{p=-\infty}^{+\infty}\sum_{q=-\infty}^{+\infty}\left(x_{k,u}^{p,q}\right)^2\right]\Bigg|_{\Phi=\hat{\Phi}_{NDA}}=0 \tag{18}$$

leading to the following NDA ML phase estimator

$$\hat{\phi}_{u,NDA}=-\frac{1}{2}\tan^{-1}\frac{\Im\left[\sum_{n=-\infty}^{+\infty}(y_u^n)^2+\sum_{\substack{k=1\\k\neq u}}^{N_u}exp\left(-2j\hat{\phi}_{k,NDA}\right)\sum_{p=-\infty}^{+\infty}\sum_{q=-\infty}^{+\infty}\left(x_{k,u}^{p,q}\right)^2\right]}{\Re\left[\sum_{n=-\infty}^{+\infty}(y_u^n)^2+\sum_{\substack{k=1\\k\neq u}}^{N_u}exp\left(-2j\hat{\phi}_{k,NDA}\right)\sum_{p=-\infty}^{+\infty}\sum_{q=-\infty}^{+\infty}\left(x_{k,u}^{p,q}\right)^2\right]} \tag{19}$$

Like with the DA estimator, coupling effects appear through correlation factors $x_{k,u}^{p,q}$.

4.2 Variance of the estimator - Computational results

In order to determine a more realistic lower bound on the phase variance than the Cramer-Rao bound, the method outlined in [4] is applied. After heavy calculations, an analytical expression of the variance is finally derived for a 2-user system.

To allow some comparison with similar performance in mono-user systems [5], the variance expression is simulated in an asynchronous system ($x_{u,v}^{n,n}=0.1x_{u,u}^{n,n}$) (Fig.2). These simulations are lead regardless of possible improvement due to code cross-correlation properties. The width of the observation window is the same as that mentioned in [5].

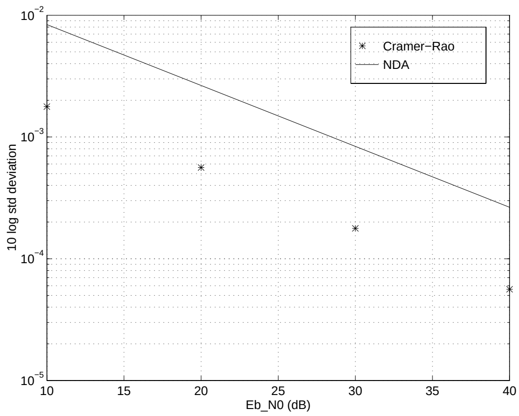

Figure 2: Lower bound on the performance

Moreover, the resistance in a Near-Far context is tested (Fig.3). The channel is the same as that used in the previous simulation, but the observation window is 10 times narrower. Regarding the Cramer-Rao bound as the ultimate performance level, reachable in a AWGN context, it logically appears that the estimator is badly performing when the power of the interfering user grows.

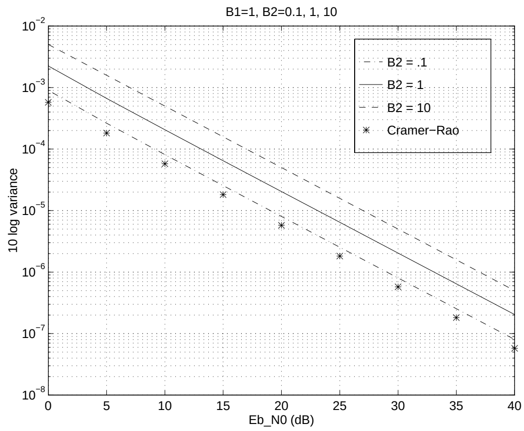

Figure 3: Near-Far effect

5 Conclusions

Maximum-Likelihood joint phase estimators for CDMA multiple access communication systems were presented, in both Data-Aided and Non Data-Aided contexts. Their analytical expression were established. Performance of these estimators were demonstrated by computing their variance and comparing them with other variances and the Cramer-Rao bound.

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